004

Name: _____ ID Number: _____

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] (a) If $\vec{u}$ and $\vec{v}$ are unit vectors, then so is $\vec{u} + \vec{v}$.

False $\vec{u} = [1, 0]$ & $\vec{v} = [0, 1]$ (both unit vectors)

but $\| \vec{u} + \vec{v} \| = \| [1, 1] \| = \sqrt{2}$

[2 marks] (b) If $\vec{u}$ and $\vec{v}$ are parallel vectors with $\vec{u}$ non-zero, then $\text{proj}_{\vec{u}}(\vec{v})$ is the zero vector.

False $\vec{u} = [1, 1]$, $\vec{v} = [2, 2]$ parallel vectors

but $\text{proj}_{\vec{u}} \vec{v} = \frac{4}{2} [1, 1] = [2, 2] \neq [0, 0]$

[3 marks] 2. Consider a code with code words in $\mathbb{Z}_6^3$ and check vector $\vec{c} = [3, 2, 1]$. Compute the check digit d that makes $[1, 2, d]$ a valid code word.

Need $\vec{v} \cdot \vec{c} \equiv 0$ in $\mathbb{Z}_6$

$[1,2,d] \cdot [3,2,1] = 3+4+d = 7+d = 1+d$ in $\mathbb{Z}_6$

So $d = 5$

[3 marks] 3. Solve the following equation for $\vec{x}$ in terms of $\vec{u}$ and $\vec{v}$:

$$\vec{x} - \vec{u} = 2(\vec{x} + 3\vec{u}) - \vec{v}$$

$\vec{x} - \vec{u} = 2\vec{x} + 6\vec{u} - \vec{v}$

$-\vec{u} - 6\vec{u} + \vec{v} = 2\vec{x} - \vec{x}$

$\vec{v} - 7\vec{u} = \vec{x}$