004

Name: ______________________ ID Number: ______________

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (that is, always true), or FALSE (that is, always false). Provide a short justification for your response.

2 marks (a) For any nonzero vectors $N_1, N_2 \in \mathbb{R}^3$, $N_1 \times N_2$ is orthogonal to $N_1 + N_2$.

True $(N_1 \times N_2) \perp N_1$ & $(N_1 \times N_2) \perp N_2$

so $(N_1 \times N_2) \cdot N_1 = 0 = (N_1 \times N_2) \cdot N_2$

Then $(N_1 \times N_2) \cdot (N_1 + N_2)$

$= (N_1 \times N_2) \cdot N_1 + (N_1 \times N_2) \cdot N_2$

$= 0 + 0$

$= 0$

2 marks (b) The planes in $\mathbb{R}^3$ with equations $2x - 5y + 3z = 5$ and $6x - 15y + 9z = 23$, respectively, are parallel.

True $\vec{n}_1 = [2, -5, 3]$

$\vec{n}_2 = [6, -15, 9]$

$\Rightarrow \vec{n}_2 = 3\vec{n}_1$ $\therefore$ parallel

3 marks

2. Find a vector equation for the line in $\mathbb{R}^3$ that passes through $[1,-1,4]$ and is orthogonal to both $[2,3,-1]$ and $[-1,2,1]$.

$\vec{x} = \vec{p} + t\,\vec{d}$ for $t \in \mathbb{R}$.

$\vec{d} = [2,3,-1] \times [-1,2,1]$
$= [5,-1,7]$

so $\vec{x} = [1,-1,4] + t[5,-1,7]$

3 marks

3. Solve the following system of linear equations:

$$2x - 3y = 3$$
$$4x + 3y = 8$$

$$\begin{array}{cc} x & y \end{array}$$
$$\left[\begin{array}{cc|c} 2 & -3 & 3 \\ 4 & 3 & 8 \end{array}\right] \to \left[\begin{array}{cc|c} 2 & -3 & 3 \\ 0 & 9 & 2 \end{array}\right]$$

$$\to \left[\begin{array}{cc|c} 1 & -3/2 & 3/2 \\ 0 & 1 & 2/9 \end{array}\right]$$

$$\to \left[\begin{array}{cc|c} 1 & 0 & 11/6 \\ 0 & 1 & 2/9 \end{array}\right]$$

so $x = 11/6$, $y = 2/9$