004

Name: ____________________ ID Number: ____________

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] (a) Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors in $\mathbb{R}^3$. If $\vec{w} \in \text{span}(\vec{u}, \vec{v})$, then $\vec{w} = \vec{u} + \vec{v}$.

False $\vec{w} = \vec{u} - \vec{v}$ is in the span$(\vec{u}, \vec{v})$ but $\vec{w} \neq \vec{u} + \vec{v}$ as long as $\vec{v} \neq \vec{0}$

so $\vec{w}$ does not have to be $\vec{u} + \vec{v}$

[2 marks] (b) Let $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$ be solutions to the homogeneous system of linear equations

$$\begin{cases} x \quad -y \quad +z = 0 \\ \quad\ \ 2y \quad -2z = 0 \end{cases}$$

If $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$, then $\vec{w}$ is also a solution of the system.

True Solving the system:

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & -2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

So all solutions to the system have the form $t\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, t \in \mathbb{R}$

If $\vec{u}$ & $\vec{v}$ are solutions, $\vec{u} = t_1\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v} = t_2\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ for $t_1, t_2 \in \mathbb{R}$

Since $\vec{w}$ is a linear combo of $\vec{u}, \vec{v}$,

$$\vec{w} = a\left(t_1\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) + b\left(t_2\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) \text{ for some } a, b \in \mathbb{R}$$

$$= (at_1 + bt_2)\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

So $\vec{w}$ is a solution.

[3 marks] 2. Compute the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ -2 & 0 & 5 \\ 1 & 1 & -4 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 3 & 7 \\ -2 & 0 & 5 \\ 1 & 1 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 7 \\ 0 & 6 & 19 \\ 0 & 2 & 11 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 7 \\ 0 & 6 & 19 \\ 0 & 0 & 14 \end{bmatrix}$$

$\operatorname{rank} A = 3$

[3 marks] 3. Let $\vec{p} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$, $\vec{q} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. Determine whether the lines $\vec{x} = \vec{p} + t\vec{u}$ and $\vec{x} = \vec{q} + s\vec{v}$ intersect and, if so, find the point of intersection.

If they intersect, $\vec{p} + t\vec{u} = \vec{q} + s\vec{v} \Rightarrow \vec{p} - \vec{q} = s\vec{v} - t\vec{u}$

so $\begin{bmatrix} -2 \\ 4 \\ 4 \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} - t\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

so $\begin{array}{c} \begin{array}{cc} s & t \end{array} \\ \left[\begin{array}{cc|c} 0 & 1 & -2 \\ 1 & -1 & 4 \\ 2 & 0 & 4 \end{array}\right] \end{array} \Rightarrow t = 2,\ s = 2$

Sub back into $\vec{x} = \vec{p} + t\vec{u} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

P.O.I.