

Name: _____ ID Number: _____

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (i.e. always true) or FALSE (i.e. not always true). Provide a short justification for your response.

[2 marks]

- (a) Let \vec{v}_1 , \vec{v}_2 and \vec{v}_3 be linearly dependent vectors in \mathbb{R}^3 . Then $\vec{v}_3 \in \text{span}(\vec{v}_1, \vec{v}_2)$.

False

$$\vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{Then } -2\vec{v}_1 + \vec{v}_2 + 0\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

so $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent

but $\vec{v}_3 \notin \text{span}(\vec{v}_1, \vec{v}_2)$

[2 marks]

- (b) Let A be a 3×2 matrix and B be a 3×3 matrix. Then $BA - 2A$ is a 3×2 matrix.

True

$$BA \quad 3 \times 2$$

$$2A \quad 3 \times 2$$

$$BA - 2A = 3 \times 2$$

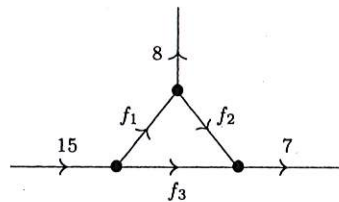
- [3 marks] 2. Let $\vec{v}_1 = [2, 4, 5]$, $\vec{v}_2 = [-1, 1, -4]$, and $\vec{v}_3 = [1, 11, -2]$ be vectors in \mathbb{R}^3 . Find the rank of the matrix $A = \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \end{bmatrix}$ and determine whether the row vectors \vec{v}_1 , \vec{v}_2 , and \vec{v}_3 are linearly independent.

$$A = \begin{bmatrix} 2 & 4 & 5 \\ -1 & 1 & -4 \\ 1 & 11 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 11 & -2 \\ 0 & 12 & -6 \\ 0 & 6 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 11 & -2 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Rank } A = 2$$

\therefore the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are lin ind since there is a zero row in REF

- [3 marks] 3. Consider the system of water pipes to the right. Set up and solve a system of linear equations to find the possible flows (positive or negative allowed, so no need to deal with inequalities).



$$\begin{aligned} 15 &= f_1 + f_3 \\ 8 + f_2 &= f_1 \\ f_2 + f_3 &= 7 \end{aligned} \rightarrow \begin{array}{ccc|c} f_1 & f_2 & f_3 & \\ \hline 1 & 0 & 1 & 15 \\ 1 & 1 & 0 & 8 \\ 0 & 1 & 1 & 7 \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 15 \\ 0 & 1 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 15 \\ 8 \\ 0 \end{bmatrix}$$

back sub :

$$\begin{aligned} f_3 &= t \\ f_2 &= 7 - t \\ f_1 &= 15 - t \end{aligned}$$