

Name: _____ ID Number: _____
(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks]

(a) Let k be a real number and I be the 2×2 identity matrix. Then $\det(kI) = k$.

False $kI = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$$\det(kI) = k^2 \neq k \text{ unless } k=0 \text{ or } 1$$

[2 marks]

(b) Let A and B be square matrices. If neither A nor B is symmetric, then $A - B$ is not symmetric.

False $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = B$ not sym. but
 $A - B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ sym.

2. Let $A = \begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 \\ 1 & -1 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$.

[3 marks]

(a) Compute the inverse of the matrix A .

$$\det A = 1 \neq 0$$

$$\begin{aligned} A^{-1} &= \frac{1}{\det A} \begin{bmatrix} -3 & 1 \\ -4 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 1 \\ -4 & 1 \end{bmatrix} \end{aligned}$$

[3 marks]

(b) Solve the following matrix equation for X :

$$AX = 3AB^T + CA.$$

$$A^{-1}AX = A^{-1}3AB^T + A^{-1}CA$$

$$X = 3B^T + A^{-1}CA$$

$$X = 3 \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}$$

$$X = \begin{bmatrix} -49 & 41 \\ -73 & 49 \end{bmatrix}$$