)04

Name: ______________________________ ID Number: ______________

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] (a) Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be vectors in $\mathbb{R}^3$. Then the dimension of span$(\vec{u}, \vec{v}, \vec{w})$ is 3.

<u>False</u> If $\vec{u}=\vec{v}=\vec{w}=\begin{bmatrix}1\\0\\0\end{bmatrix}$

Then $\text{span}(\vec{u},\vec{v},\vec{w}) = \text{span}\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)$

so a basis for $\text{span}(u,v,w) = \left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}$

$\therefore$ dim. $= 1$

[2 marks] (b) The map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x^2-y\\3x+2y\end{bmatrix}$ is a linear transformation.

<u>False</u> $T\left(\begin{bmatrix}1\\1\end{bmatrix}+\begin{bmatrix}2\\3\end{bmatrix}\right) = T\left(\begin{bmatrix}3\\4\end{bmatrix}\right) = \begin{bmatrix}9-4\\9+8\end{bmatrix} = \begin{bmatrix}5\\17\end{bmatrix}$

but $T\begin{bmatrix}1\\1\end{bmatrix} + T\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}0\\5\end{bmatrix}+\begin{bmatrix}1\\12\end{bmatrix} = \begin{bmatrix}1\\17\end{bmatrix}$ $\neq$

$\therefore$ Not linear transf.

[3 marks] 2. Let $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ and $T_2 : \mathbb{R}^2 \to \mathbb{R}^3$ be linear transformations defined by

$$T_1 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+2y \\ -2x-4y \end{bmatrix} \quad \textit{and} \quad T_2 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 5x+4y \\ x-y \end{bmatrix},$$

respectively. Find the standard matrix $[T_2 \circ T_1]$ of the composition $T_2 \circ T_1$.

$[T_1] = \begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$ & $[T_2] = \begin{bmatrix} 2 & 0 \\ 5 & 4 \\ 1 & -1 \end{bmatrix}$

so $[T_2 \circ T_1] = [T_2][T_1] = \begin{bmatrix} 2 & 4 \\ -3 & -6 \\ 3 & 6 \end{bmatrix}$

[3 marks] 3. Let $A = \begin{bmatrix} 1 & -2 & 5 \\ 3 & -4 & 7 \end{bmatrix}$ be a matrix and $T_A : \mathbb{R}^3 \to \mathbb{R}^2$ be the corresponding matrix transformation. Find a nonzero vector $\vec{x}$ in $\mathbb{R}^3$ such that $T_A(\vec{x}) = \vec{0}$.

$\begin{bmatrix} 1 & -2 & 5 \\ 3 & -4 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

solve the system

$\left[\begin{array}{ccc|c} 1 & -2 & 5 & 0 \\ 3 & -4 & 7 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & -4 & 0 \end{array}\right]$

Let $x_3 = t$, $x_2 = 4t$, $x_1 = 3t$

so solutions: $t\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$