Name: _____ ID Number: _____

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] (a) Let A and B be two $n \times n$ matrices. Then $det(A+B) = det(A) + det(B)$.

False Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then $\det(A+B) = \det\begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix} = 4$

but $\det A + \det B = -1$ $\not=$

[2 marks] (b) Let $A = \begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$ be a matrix. Then $\lambda = -3$ is an eigenvalue of A.

True λ is an e-value if $A\vec{x} = \lambda\vec{x}$ has non-zero solution $\vec{x}$

so we need to solve $\vec{0} = (\lambda I - A)\vec{x}$

$\Rightarrow$

$$\begin{bmatrix} -4 & -2 & | & 0 \\ 2 & 1 & | & 0 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

has solution $\vec{x} = t\begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$

$\therefore$ It is an e-value

[3 marks] 2. Let $P = \begin{bmatrix} 0.3 & 0.5 \\ 0.7 & 0.5 \end{bmatrix}$ be the transition matrix for a Markov chain with two states. Find all steady state vectors of the Markov chain.

Need $P\vec{x} = \vec{x}$ so need to solve $\vec{0} = (I - P)\vec{x}$

$$\Rightarrow \left[\begin{array}{cc|c} 0.7 & -0.5 & 0 \\ -0.7 & 0.5 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -5/7 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

So $\vec{x} = t\begin{bmatrix} 5/7 \\ 1 \end{bmatrix}$

[3 marks] 3. Let $A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 2 & 5 \\ 1 & 3 & 3 \end{bmatrix}$ be a 3×3 matrix. Compute the determinant of A.

$$\det A = 2\det\begin{bmatrix} 2 & 5 \\ 3 & 3 \end{bmatrix} - 0 + \det\begin{bmatrix} -1 & -1 \\ 2 & 5 \end{bmatrix}$$

$$= -21$$