004

Name: ______________________________ ID Number: ______________

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] (a) Let A and B be $n \times n$ matrices. If A is not invertible, then AB is not invertible.

True If A is not invertible then $\det A = 0$

so $\det AB = (\det A)(\det B)$

$= 0 \cdot (\det B)$

$= 0$

$\therefore AB$ not invertible

[2 marks] (b) Let A and B be $n \times n$ matrices. If λ is an eigenvalue of both A and B, then it is also an eigenvalue of $A + B$.

False If $A = B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $\lambda = 1$ is an e-value of A & B

But $A+B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ only has e-value $\lambda = 2$

2. Let $A = \begin{bmatrix} 5 & 0 & 6 \\ -8 & 1 & -12 \\ -1 & 0 & 0 \end{bmatrix}$ be a 3×3 matrix.

[3 marks] (a) Show that $\lambda = 2$ is an eigenvalue of A, and find the corresponding eigenspace.

$$\det(A-\lambda I) = \det(A-2I) = \det\left(\begin{bmatrix} 3 & 0 & 6 \\ -8 & -1 & -12 \\ -1 & 0 & -2 \end{bmatrix}\right)$$
$$= (-1)\det\begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix} = 0$$

so $\lambda=2$ is an e-value

$E_2 = \text{null}(A-2I)$

$$\left[\begin{array}{ccc|c} 3 & 0 & 6 & 0 \\ -8 & -1 & -12 & 0 \\ -1 & 0 & -2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \vec{x} = t\begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix},$$

$$\therefore E_2 = \text{span}\left\{\begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}\right\}.$$

[3 marks] (b) Compute the characteristic polynomial of A and find all eigenvalues of A.

$$\text{char. poly} = \det(A-\lambda I)$$
$$= \det\begin{bmatrix} 5-\lambda & 0 & 6 \\ -8 & 1-\lambda & -12 \\ -1 & 0 & -\lambda \end{bmatrix}$$
$$= (1-\lambda)\det\begin{bmatrix} 5-\lambda & 6 \\ -1 & -\lambda \end{bmatrix}$$
$$= (1-\lambda)(\lambda^2 - 5\lambda + 6)$$
$$= (1-\lambda)(\lambda-2)(\lambda-3)$$

$\therefore$ e-values $= \lambda = 1, 2, 3.$