004.

Name: ______________________ ID Number: ____________

(Please Print)

1. Determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] Let A be a 3×3 matrix. If A has 3 distinct eigenvalues, then it is invertible.

False If 0 is one e-value then it is not invertible.

say $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, has e-values $\lambda = 0, 1, 2$

but $\det A = 0 \Rightarrow$ not inv.

2. Let $A = \begin{bmatrix} 7 & 18 & -9 \\ -3 & -8 & 3 \\ 0 & 0 & -2 \end{bmatrix}$ be a 3×3 matrix.

The characteristic polynomial of A is $-\lambda^3 - 3\lambda^2 + 4$.

2

[~~3~~ marks] (a) Show that $\begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$ is an eigenvector of A, and find the corresponding eigenvalue.

$$\begin{bmatrix} 7 & 18 & -9 \\ -3 & -8 & 3 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$

it is an e-vector for $\lambda = 1$

4

[~~3~~ marks] (b) Find the other eigenvalues of A and bases for the corresponding eigenspaces.

$$\text{char poly} = -\lambda^3 - 3\lambda^2 + 4$$
$$= -(\lambda-1)(\lambda+2)^2$$

$$E_{-2} = \left[\begin{array}{ccc|c} 9 & 18 & -9 & 0 \\ -3 & -6 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\text{basis}: \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$E_1 = \left[\begin{array}{ccc|c} 6 & 18 & -9 & 0 \\ -3 & -9 & 3 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\text{basis}: \left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

[2 marks] (c) Determine whether or not A is diagonalisable, and if it is, find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

Diagonalizable ∵ the sum of geo. mult = 3

$$P = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$