

Name: _____ ID Number: _____

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks]

- (a) If \vec{u} and \vec{v} are parallel, then the projection of \vec{v} onto \vec{u} is equal to \vec{u} .

F

$$\vec{u} = [1, 1] \quad \vec{v} = [2, 2] \quad \text{parallel}$$

$$\text{but } \text{proj}_{\vec{u}}(\vec{v}) = \frac{1}{2}[1, 1] = [0.5, 0.5] \neq \vec{u}$$

[2 marks]

- (b) If $\vec{u} \cdot \vec{v} < 0$, then the angle between \vec{u} and \vec{v} is acute (less than 90°).

F

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$$

$$\text{If } \vec{u} \cdot \vec{v} < 0 \Rightarrow \|\vec{u}\| \|\vec{v}\| \cos \theta < 0$$

$$\Rightarrow \cos \theta < 0 \quad (\text{since } \|\vec{u}\|, \|\vec{v}\| \geq 0)$$

$$\Rightarrow 90^\circ < \theta < 180^\circ$$

- [3 marks] 2. Consider a code with code words in \mathbb{Z}_6^3 and check vector $\vec{c} = [3, 2, 1]$. Compute the check digit d that makes $[1, 2, d]$ a valid code word.

$$\text{Need } \vec{v} \cdot \vec{c} = 0 \text{ in } \mathbb{Z}_6$$

$$[1, 2, d] \cdot [3, 2, 1] = 3 + 4 + d = 7 + d = 1 + d \text{ in } \mathbb{Z}_6$$

$$\Rightarrow d = 5$$

- [3 marks] 3. Express $\vec{w} = [2, 3]$ as a linear combination of $\vec{u} = [1, 1]$ and $\vec{v} = [-1, 0]$.

$$\vec{w} = [2, 3] = [3, 3] + [-1, 0]$$

$$\Rightarrow \vec{w} = 3\vec{u} + \vec{v}$$