005

Name: ______________________ ID Number: ______________________

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (that is, always true), or FALSE (that is, always false). Provide a short justification for your response.

2 marks (a) For any real numbers a, b, c, d, the set of solutions to the system of linear equations

$$ax + by = 0$$
$$cx + dy = 0$$

is infinite.

F for $a=1, b=0, c=0, d=1$
we have $x=0, y=0$ (unique sol^{n})

2 marks (b) The vector $[1, 3, 2]$ is a direction vector for the line of intersection of the planes with equations $2x - 4y + 3z = 3$ and $7x + 3y - 5z = 2$, respectively.

F the line of intersection lies on both planes
So the direction vector of this line is $\perp$ to
the normals of both plane
but $[1,3,2] \cdot [2,-4,3] \neq 0$
& $[1,3,2] \cdot [7,3,-5] \neq 0$

3 marks

2. Find a vector equation for the line in $\mathbb{R}^3$ that passes through both $[1, -1, 3]$ and $[2, 3, -1]$.

$\vec{x} = \vec{p} + t\vec{d}$ for $t \in \mathbb{R}$

$\vec{d} = [2, 3, -1] - [1, -1, 3]$

$= [1, 4, -4]$

so $\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ 4 \\ -4 \end{bmatrix}$

3 marks

3. Find the distance from the point $Q = [1, 3, 1]$ to the plane with equation $2x + 4y + z = 2$.

$A = [1, 0, 0]$ is a point on plane.

so $\vec{AQ} = [0, 3, 1]$

$\vec{n} = [2, 4, 1]$

The distance $= \| \mathrm{Proj}_{\vec{n}} \vec{AQ} \| = \| \frac{13}{21} [2, 4, 1] \|$

$= \left(\frac{13}{21}\right) \sqrt{21}$.