005

Name: ______________________________ ID Number: ______________

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] (a) Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^3$. Then $\vec{v} \in \text{span}(\vec{u} - \vec{v}, \vec{u})$.

T $\vec{v} = -(\vec{u} - \vec{v}) + \vec{u}$

so $\vec{v} \in \text{span}(\vec{u} - \vec{v}, \vec{u})$

[2 marks] (b) Let $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^2$ be solutions to the system of linear equations

$$\begin{cases} x - 3y &= -2 \\ x + y &= 2 \end{cases}$$

If $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$, then $\vec{w}$ is also a solution of the system.

F solving:

$$\left[\begin{array}{cc|c} 1 & -3 & -2 \\ 1 & 1 & 2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right]$$

only solution is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

If $\vec{u}, \vec{v}$ are solutions, $\vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \vec{v}$

$\vec{w} = \vec{u} + \vec{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ is a lin combo of $\vec{u}, \vec{v}$,

but $\vec{w}$ is not a solution to the sys.

[3 marks] 2. Compute the rank of the matrix $A = \begin{bmatrix} 1 & -3 & 7 \\ 0 & 4 & -5 \\ 2 & 1 & 7 \end{bmatrix}$.

$$\begin{bmatrix} 1 & -3 & 7 \\ 0 & 4 & -5 \\ 2 & 1 & 7 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 7 \\ 0 & 4 & -5 \\ 0 & 7 & -7 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 7 \\ 0 & 1 & -5/4 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & -3 & 7 \\ 0 & 1 & -5/4 \\ 0 & 0 & 1/4 \end{bmatrix}$$

$\text{Rank}\, A = 3$

[3 marks] 3. Let $\vec{p} = \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$, $\vec{q} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix}$. Determine whether the lines $\vec{x} = \vec{p} + t\vec{u}$ and $\vec{x} = \vec{q} + s\vec{v}$ intersect and, if so, find the point of intersection.

Need $\vec{p} + t\vec{u} = \vec{q} + s\vec{v} \Rightarrow \vec{p} - \vec{q} = s\vec{v} - t\vec{u}$

$\Rightarrow \begin{bmatrix} 0 \\ -4 \\ 2 \end{bmatrix} = s\begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix} - t\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Solve

$$\begin{array}{cc} s & t \end{array}$$
$$\left[\begin{array}{cc|c} -3 & -1 & 0 \\ 2 & 0 & -4 \\ 2 & 1 & 2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{array}\right]$$ so $s = -2$, $t = 6$

Sub back into $\vec{x} = \vec{p} + t\vec{u}$

so $\vec{x} = \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} + 6\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \\ -5 \end{bmatrix}$