Section 005

Name: ______________________________ ID Number: ____________

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

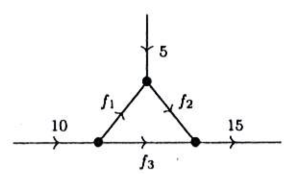
[2 marks] (a) Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^3$. Then span$(\vec{u}, \vec{v})$ is a plane through the origin in $\mathbb{R}^3$.

F $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \vec{v}$ then $\text{Span}(\vec{u}, \vec{v}) = \text{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$

$= \left\{ \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix} \middle| x \in \mathbb{R} \right\}$

$=$ x-axis not a plane

[2 marks] (b) Consider the network to the right.
If $f_3 \leqslant 1$, then $f_1 \geqslant 9$.



T $10 = f_1 + f_3$

$5 + f_1 = f_2$

$f_2 + f_3 = 15$

$$\Rightarrow \begin{array}{c} \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 1 & 10 \\ -1 & 1 & 0 & 5 \\ 0 & 1 & 1 & 15 \end{array}\right] \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 10 \\ 0 & 1 & 1 & 15 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $f_3 = t$ then $f_2 + f_3 = 15$, $\cancel{f_2}$ $f_1 + f_3 = 10$

$\Rightarrow f_3 = t$, $f_2 = 15 - t$, $f_1 = 10 - t$

If $f_3 \leq 1$ then $f_1 = 10 - t \geq 9$.

[3 marks] 2. Determine whether the vectors $\vec{v}_1 = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix}$ are linearly dependent and, if so, find a dependence relation among the vectors.

$$\begin{array}{ccc} a & b & c \end{array}$$

$$\left[\begin{array}{ccc|c} 4 & 2 & 6 & 0 \\ 1 & 1 & 1 & 0 \\ 3 & 5 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

so $a = -2t \quad b = t \quad c = t$

$\Rightarrow \vec{v}_1, \vec{v}_2, \vec{v}_3$ lin dep.

$-2t\vec{v}_1 + t\vec{v}_2 + t\vec{v}_3 = \vec{0}$

[3 marks] 3. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \\ 3 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} -2 & 1 \\ -2 & 2 \end{bmatrix}$. Compute $AB + 3C$.

$$AB + 3C = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \end{pmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \\ 3 & 0 \end{bmatrix} + 3\begin{bmatrix} -2 & 1 \\ -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 1 \\ 11 & 0 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ -6 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$