

Name: _____ ID Number: _____
(Please Print)

1. For each statement below, determine whether the given statement is TRUE (i.e. always true) or FALSE (i.e. not always true). Provide a short justification for your response.

[2 marks]

(a) Let k be a nonzero real number and I be the identity matrix of size n . Then the matrix $\frac{1}{k}I$ is invertible.

True $(\frac{1}{k}I)(kI) = I = (kI)(\frac{1}{k}I)$
 $\Rightarrow (\frac{1}{k}I)^{-1} = (kI)$. so invertible

[2 marks]

(b) Let A be an invertible matrix. If A is symmetric, then A^{-1} is symmetric too.

True $(A^{-1})^T = (A^T)^{-1}$ (property of transpose & inverse)
 $= A^{-1}$ since $A^T = A$
 $\therefore A^{-1}$ symmetric.

2. Let $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix}$.

[3 marks]

(a) Compute the inverse of the matrix A .

$$\det A = 5 \neq 0$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

[3 marks]

(b) Solve the following matrix equation for X :

$$AX = (B + 2A)A.$$

$$A^{-1}AX = A^{-1}(B+2A)A$$

$$X = A^{-1}BA + A^{-1}2AA$$

$$X = A^{-1}BA + 2A$$

$$X = \begin{bmatrix} -3 & -9 \\ -1 & -3 \end{bmatrix}$$