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Name: ______________________ ID Number: ____________
(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] (a) Let S be a subspace of $\mathbb{R}^3$. Suppose that $\{\vec{u}_1, \vec{u}_2\}$ and $\{\vec{v}_1, \vec{v}_2\}$ are two bases for S. Then $\{\vec{u}_1, \vec{u}_2, \vec{v}_1, \vec{v}_2\}$ is also a basis for S.

False let s be plane $= t\begin{bmatrix}1\\0\\0\end{bmatrix} + s\begin{bmatrix}0\\1\\0\end{bmatrix}$ (x-y plane)

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then $\{\vec{u}_1, \vec{u}_2\} = \left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$
& $\{\vec{v}_1, \vec{v}_2\} = \left\{\begin{bmatrix}2\\0\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}\right\}$ are 2 bases for S.

but $\{\vec{u}_1, \vec{v}_2, \vec{v}_1, \vec{v}_2\} = \left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}2\\0\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}\right\}$ is not lin ind $\therefore$ not a basis

$\boxed{\text{or}}$ set 4 vectors in $\mathbb{R}^3$ cannot be lin ind $\therefore$ not basis.

[2 marks] (b) The map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x-2y\\3y+1\end{bmatrix}$ is a linear transformation.

$\begin{bmatrix}u\\v\end{bmatrix}, \begin{bmatrix}a\\b\end{bmatrix} \in \mathbb{R}^2$.

$T\left(\begin{bmatrix}u\\v\end{bmatrix} + \begin{bmatrix}a\\b\end{bmatrix}\right) = T\left(\begin{bmatrix}u+a\\v+b\end{bmatrix}\right)$
$= \begin{bmatrix}u+a-2v-2b\\3v+3b+1\end{bmatrix}$

$T\begin{bmatrix}u\\v\end{bmatrix} + T\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}u-2v\\3v+1\end{bmatrix} + \begin{bmatrix}a-2y\\3b+1\end{bmatrix}$ $\neq$.
$= \begin{bmatrix}u+a-2v-2y\\3v+3b+2\end{bmatrix}$

$\boxed{\text{or}}$ $T\left(\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}2\\3\end{bmatrix}\right) = T\left(\begin{bmatrix}3\\4\end{bmatrix}\right) = \begin{bmatrix}3-8\\12+1\end{bmatrix} = \begin{bmatrix}-5\\13\end{bmatrix}$

$T\begin{bmatrix}1\\1\end{bmatrix} + T\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}1-2\\3+1\end{bmatrix} + \begin{bmatrix}2-6\\9+1\end{bmatrix} = \begin{bmatrix}-5\\14\end{bmatrix}$ $\neq$

[3 marks] 2. Let $T_1 : \mathbb{R}^3 \to \mathbb{R}^2$ and $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ be linear transformations defined by

$$T_1 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3x + 2y \\ y - z \end{bmatrix} \quad \textit{and} \quad T_2 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x - 2y \\ x \end{bmatrix},$$

respectively. Find the standard matrix $[T_2 \circ T_1]$ of the composition $T_2 \circ T_1$.

$[T_1] = \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ & $[T_2] = \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow [T_2 \circ T_1] = \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 2 \\ 3 & 2 & 0 \end{bmatrix}$

or

$(T_2 \circ T_1) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = T_2 \begin{bmatrix} 3x+2y \\ y-z \end{bmatrix} = \begin{bmatrix} 3x+2y-2y+2z \\ 3x+2y \end{bmatrix} = \begin{bmatrix} 3x+2z \\ 3x+2y \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 & 2 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

[3 marks] 3. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ be a matrix and $T_A : \mathbb{R}^2 \to \mathbb{R}^3$ be the corresponding matrix transformation. Let $\vec{b} = \begin{bmatrix} -1 \\ -1 \\ -3 \end{bmatrix}$ be a vector in $\mathbb{R}^3$. Find a vector $\vec{x}$ in $\mathbb{R}^2$ such that $T_A(\vec{x}) = \vec{b}$.

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$\begin{bmatrix} 1 & 2 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cancel{x_3} \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -3 \end{bmatrix}$

$\Rightarrow \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 2 & 5 & -1 \\ 3 & 6 & -3 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$

$\Rightarrow x_1 + 2x_2 = -1$
$x_2 = 1$
$\Rightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

check $\begin{bmatrix} 1 & 2 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3+2 \\ -6+5 \\ -9+6 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -3 \end{bmatrix}$ ✓.