005

Name: _____ ID Number: _____

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] (a) Let A be a square matrix. Then $det(2A) = 2det(A)$.

F let $A=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ Then $\det(2A)=\det\begin{bmatrix}2&0\\0&2\end{bmatrix}$

$=4$

but $2\det A = 2$

[2 marks] (b) Let $A = \begin{bmatrix}2&2\\3&1\end{bmatrix}$ be a matrix. Then $\vec{v} = \begin{bmatrix}1\\2\end{bmatrix}$ is an eigenvector of A.

F $A\vec{v} = \begin{bmatrix}2&2\\3&1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}6\\5\end{bmatrix}$

$\begin{bmatrix}6\\5\end{bmatrix}$ is not a scalar multiple of $\begin{bmatrix}1\\2\end{bmatrix}$

$\therefore \vec{v}$ is not an e-vector

[3 marks] 2. Let $P = \begin{bmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{bmatrix}$ be the transition matrix for a Markov chain with two states.

Find all steady state vectors of the Markov chain.

Need $P\vec{x} = \vec{x}$ so solve $\vec{0} = (I - P)\vec{x}$

$\Rightarrow$ solve $\begin{bmatrix} 0.4 & -0.1 & 0 \\ -0.4 & 0.1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1/4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \vec{x} = t\begin{bmatrix} 1/4 \\ 1 \end{bmatrix}$

[3 marks] 3. Let $A = \begin{bmatrix} 2 & 0 & -1 \\ -2 & -4 & 5 \\ 3 & 3 & -3 \end{bmatrix}$ be a 3×3 matrix. Compute the determinant of A.

$\det A = 2\det\begin{bmatrix} -4 & 5 \\ 3 & -3 \end{bmatrix} - 0 + (-1)\det\begin{bmatrix} -2 & -4 \\ 3 & 3 \end{bmatrix}$

$= 2(12)$