005

Name: ______________________________ ID Number: ______________

(Please Print)

1. For each statement below, determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] (a) Let A and B be invertible $n \times n$ matrices. Then AB is invertible.

T If A, B invertible, $\det A \neq 0$ & $\det B \neq 0$

$\Rightarrow \det(AB) = (\det A)(\det B) \neq 0.$

[2 marks] (b) Let A and B be $n \times n$ matrices. If $\vec{v}$ is an eigenvector of both A and B, then it is also an eigenvector of $A + B$.

T we know $A\vec{v} = \lambda_1 \vec{v}$ & $B\vec{v} = \lambda_2 \vec{v}$

$$\Rightarrow (A+B)\vec{v} = A\vec{v} + B\vec{v}$$
$$= \lambda_1 \vec{v} + \lambda_2 \vec{v}$$
$$= (\lambda_1 + \lambda_2)\vec{v}$$

$\therefore$ "it is an e-vector of $A+B$.

2. Let $A = \begin{bmatrix} 0 & 0 & -6 \\ 2 & 1 & 12 \\ 1 & 0 & 5 \end{bmatrix}$ be a 3×3 matrix.

[3 marks] (a) Show that $\lambda = 3$ is an eigenvalue of A, and find the corresponding eigenspace.

$\det(A-\lambda I) = \det(A-3I) = \det \begin{bmatrix} -3 & 0 & -6 \\ 2 & -2 & 12 \\ 1 & 0 & 2 \end{bmatrix}$
$= 0$

$\therefore \lambda = 3$ is an e-value.

$E_3 = \text{null}(A-3I)$

solve $\left[\begin{array}{ccc|c} -3 & 0 & -6 & 0 \\ 2 & -2 & 12 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\therefore \vec{x} = t\begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}$

$\therefore E_3 = \text{span}\left\{\begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}\right\}$

[3 marks] (b) Compute the characteristic polynomial of A and find all eigenvalues of A.

charpoly: $\det(A-\lambda I)$

$= \det \begin{bmatrix} -\lambda & 0 & -6 \\ 2 & 1-\lambda & 12 \\ 1 & 0 & 5-\lambda \end{bmatrix}$

$= (1-\lambda)\det \begin{bmatrix} -\lambda & -6 \\ 1 & 5-\lambda \end{bmatrix}$

$= (1-\lambda)(\lambda^2 - 5\lambda + 6)$

$= (1-\lambda)(\lambda-2)(\lambda-3)$

$\therefore$ e-values are $\lambda = 1, 2, 3$