005

Name: ______________________ ID Number: ______________

(Please Print)

1. Determine whether the given statement is TRUE (*i.e.* always true) or FALSE (*i.e.* not always true). Provide a short justification for your response.

[2 marks] Let A be a 3×3 matrix. If $\vec{v}_1$ and $\vec{v}_2$ are two linearly independent eigenvectors of A, then they correspond to different eigenvalues.

F $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ e-values are $\lambda = 1, 2$

$E_1 = \text{null}(A - I) = \text{null} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

So $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ are lin-ind e-vectors corresponding to $\lambda = 1$

2. Let $A = \begin{bmatrix} -3 & -12 & 6 \\ 2 & 7 & -2 \\ 0 & 0 & 3 \end{bmatrix}$ be a 3×3 matrix.

The characteristic polynomial of A is $-\lambda^3 + 7\lambda^2 - 15\lambda + 9$.

[4 marks] (a) Show that 3 is an eigenvalue of A, and find a basis for the corresponding eigenspace. Determine the algebraic and geometric multiplicities of 3.

$\det(A - 3I) =$ char poly evaluated at $\lambda = 3$

$= -(3)^3 + 7(3)^2 - 15(3) + 9 = 0$

$\therefore$ 3 is an e-value

$E_3 = \text{null}(A - 3I)$

solve $\left[\begin{array}{ccc|c} -6 & -12 & 6 & 0 \\ 2 & 4 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$E_3 = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

Also $-\lambda^3 + 7\lambda^2 - 15\lambda + 9 = -(\lambda - 1)(\lambda - 3)^2$

so 3 has alg mult 2 & geo mult 2.

[2 marks] (b) Find the other eigenvalues of A and bases for the corresponding eigenspaces.

(see factored form in a)

$\lambda = 1$ is an e-value

$E_1 = \text{null}(A - I)$

solve $\left[\begin{array}{ccc|c} -4 & -12 & 6 & 0 \\ 2 & 6 & -2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$E_1 = \text{Span}\left\{\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}\right\}$.

[2 marks] (c) Determine whether or not A is diagonalisable, and if it is, find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

Diagonalizable $\because$ total geo mult $= 3$

$P = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$