

## TOPICS FOR FINAL EXAM — M9052

Questions will be similar to homework questions and others exercises from Hatcher, so go over those. Chosen to have clean solutions. Be sure to know precise statements of theorems and definitions, and to check hypotheses of theorems used.

Legend: \* = important; – = medium; × = **not** on the final.

- Pointed and unpointed homotopy relation, homotopy equivalence, basic properties.
- $\pi_1(X, x_0)$ :
  - group structure, functor, homotopy invariance, change of basepoint
  - \* known calculations
  - \* use for proving theorems such as impossibility of retractions, def. retractions
  - × Borsuk-Ulam, fundamental theorem of algebra
  - \* van Kampen theorem: precise statement, how to use it, free groups; not proof
- Covering spaces:
  - homotopy lifting and path lifting
  - $p_*$  injective
  - number of sheets =  $[\pi_1(\text{base}) : p_*(\pi_1(\text{cover}))]$
  - lifting criterion
  - universal cover exists (but not proof)
  - \* classification of covers (pointed and unpointed) using subgroups (but not proof)
  - normal cover iff normal subgroup
  - ×  $n$ -sheeted covers biject with actions of  $\pi_1$  on a set of size  $n$
  - deck transformations
  - × covering space actions
  - \* examples, such as  $S^1 \vee S^1$ ,  $\mathbb{R}P^2$ , torus, etc.
- ×  $\pi_n$  for  $n > 1$
- \* CW complexes,  $\pi_1(\text{CW complex})$
- Homology:
  - $\Delta$ -complexes, examples
  - \* simplicial homology, example calculations
  - singular homology, unreduced and reduced
  - \* basic theorems 2.6 (disjoint unions), 2.7 ( $H_0$ ), 2.8 ( $H_*(\text{pt})$ ), 2.9 (functorial), 2.10 (homotopy invariant)
  - × prism operators
  - chain homotopy
  - \* relative homology, LES of a pair (not proof)
  - \* excision: precise statement, examples, not proof
  - good pair,  $H_n(X, A) \cong \tilde{H}_n(X/A)$  for  $(X, A)$  a good pair
  - $H_*(S^n)$
  - × invariance of dimension, local homology
  - singular agrees with simplicial (not proof)
  - × cohomology
  - × anything only covered in talks