Algebraic Topology, Homework Set 4

Due Monday, March 14 at 11:59pm. We have finished the required material, although I haven't yet given an example of computing H_*^Δ. That will come on March 8, or you can read the text.

Please justify all answers and follow the formatting guidelines from HW1.

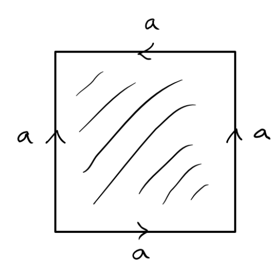
Q1 (5 marks): Let $Y \subseteq \mathbb{R}^2$ be the union of three circles of radius 1, centered at $(-2, 0)$, $(0, 0)$ and $(2, 0)$. Let the non-trivial element of $G = \mathbb{Z}/2$ act on Y by the map sending $y \in Y$ to $-y$. Check that this is a covering space action, and describe the quotient space Y/G. Is the map $Y \to Y/G$ a cover we have seen before?

Q2 (a) (3 marks): Let $X = S^1 \vee S^2$, and write a for the homotopy class going once around the left summand. Let x_0 denote the wedge point. Then $\pi_1(X, x_0) \cong \mathbb{Z}$, generated by a. Define an action of $\pi_1(X, x_0)$ on the set $\{x_1, x_2\}$ by $ax_1 = x_2$ and $ax_2 = x_1$. Describe the corresponding cover of X.

(b) (2 marks): What is the group of deck transformations of this cover? Is it a normal cover?

[Part (a) is using the material from Hatcher on pages 68–70, which I also covered briefly in class. All you need to do is to find a cover that gives rise to the specified action on $p^{-1}(x_0)$.]

Q3 (5 marks): Consider the space X formed from the square



by identifying all four edges to one edge, as shown.

(a) Describe a CW structure on X with one 0-cell, one 1-cell and one 2-cell. Using this CW structure and the CW method we learned, compute $\pi_1(X, v)$, where v is the vertex.

(b) Put a Δ-complex structure on X and use it to compute $H_*^\Delta(X)$ directly from the definition.