Algebraic Topology, Homework Set 2

Due Saturday, February 5, 11:59pm. We have covered all of the needed material (up to the end of Section 1.1).

Please **reread the instructions given on HW1** about style and formatting.

Q1 (4 marks): Let X be a space and let $x_0 \in A \subseteq X$. Show that if $\pi_1(X, x_0) \cong \mathbb{Z}$ and A is a retract of X, then $\pi_1(A, x_0)$ is either trivial or isomorphic to $\mathbb{Z}$.

Q2 (3 marks each): Show that there are no retractions $r : X \to A$ in the following cases:

(a) $X = \mathbb{R}^3 \setminus \{0\}$ and $A = \{(x, y, 0) \in \mathbb{R}^3 \mid x^2 + y^2 = 1\}$. [Undergrads only.]

(b) X the boundary of $D^2 \times I$ and $A = S^1 \times I \subseteq X$.

(c) $X = \mathbb{R}^2 \setminus \{(2, 0)\}$ and $A = S^1$.

(d) X the Möbius band and A its boundary. Here $X = (I \times I)/\sim$, where $(0, s) \sim (1, 1-s)$. [Grad students only.]

[For all parts, be careful with basepoints. The last one is tricky.]

Q3 (4 marks): Is there a retraction from $S^1 \vee S^1$ onto the left summand? Is there a deformation retraction? [Here $S^1 \vee S^1$ is the "figure eight" space we started discussing in Section 1.2. But try to do this question using only material in Section 1.1.]

Q4 (4 marks): Let Y be $\mathbb{R}^2$ with two points removed, and let $y_0 \in Y$. Show that $\pi_1(Y, y_0) \cong \pi_1(S^1 \vee S^1, x_0)$, where x_0 is the crossing point of the figure eight. [You don't need to give a formula for the homotopy you come up with, but can instead draw a careful picture.]