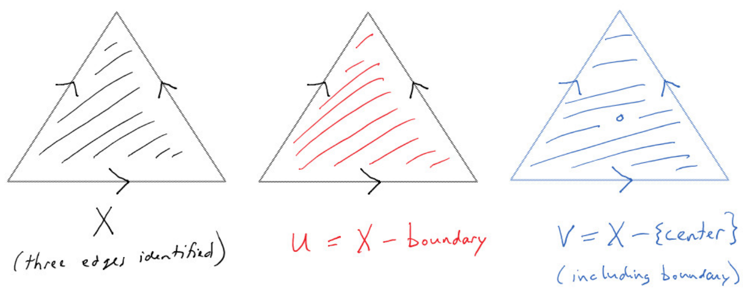
Algebraic Topology, Homework Set 3

Due Saturday, February 19 at 11:59pm. We have finished the required material. Please justify all answers and follow the formatting guidelines from HW1.

Q1 (4 marks) : Let X be the Möbius band with boundary A. Let $T = S^1 \times S^1$, and consider the subspace $S^1 \times \{1\}$. Choose a homeomorphism $\phi : A \cong S^1 \times \{1\}$, and let Y be the space formed from the disjoint union of X and T by identifying $a \in A$ with the point $\phi(a)$ in $S^1 \times \{1\}$, for each $a \in A$. Describe $\pi_1(Y, y_0)$, for y_0 one of the identified points, in terms of generators and relations, with as few generators as possible.

Q2 (4 marks): Use the Seifert-Van Kampen theorem to compute $\pi_1(X)$, where X is obtained from the triangle shown at the left by identifying the three edges to a single edge, preserving the given orientations. Subspaces U and V that will be handy are shown in the other figures. Be careful with basepoints.



Q3 (3 marks): Describe the connected cover $p : (\tilde{X}, \tilde{x}_0) \to S^1 \vee S^1$ such that $p_* \pi_1(\tilde{X}, \tilde{x}_0) = \langle b^2 \rangle$, where b denotes the homotopy class going once around the right summand of $S^1 \vee S^1$.

Q4 (4 marks): Show that there are uncountably many non-isomorphic connected covers of $S^1 \vee S^1$. (Here we are considering unpointed covers.)

Q5 (a) (2 marks): Give an example of a map $\tilde{X} \to X$ which has the unique homotopy lifting property (Prop 1.30) but which is not a covering space. (Hint: there is an example where X is a point.)

(b) (Bonus, 2 marks): Give an example where $\tilde{X}$ is path connected. (Hint: I think there is an example where $X = S^1$.)