Algebraic Topology, Homework Set 5

Due Saturday, March 26 at 11:59pm. But we have already covered the required material, so you can get it done sooner.

Please justify all answers and follow the formatting guidelines from HW1.

Q1 (5 marks): Consider the sequence

$$0 \longrightarrow \mathbb{Z}^2 \xrightarrow{\partial_2} \mathbb{Z}^2 \xrightarrow{\partial_1} \mathbb{Z} \longrightarrow 0$$

defined as follows. Write a, b for a basis for the left $\mathbb{Z}^2$, c, d for a basis for the middle $\mathbb{Z}^2$ and e for a basis for the right $\mathbb{Z}$. Define the boundary maps to be the homomorphisms such that $\partial_2(a) = 4c + 2d$, $\partial_2(b) = 4c + 4d$, $\partial_1(c) = e$ and $\partial_1(d) = -2e$.

(a) Is this a chain complex?

(b) What if $\partial_2(b) = 8c + 4d$ instead?

(c) Compute the three possibly non-trivial homology groups for whichever choice gives a chain complex.

Q2 (4 marks): Consider the chain complexes

$$A : \quad \cdots \longrightarrow 0 \longrightarrow 0 \longrightarrow \mathbb{Z} \longrightarrow 0$$

and

$$B : \quad \cdots \longrightarrow 0 \longrightarrow \mathbb{Z} \xrightarrow{2} \mathbb{Z} \longrightarrow 0$$

where the rightmost $\mathbb{Z}$ is in degree 0 in both cases.

(a) Compute the set of all chain maps $A \to B$ and the set of all chain homotopy equivalence classes of maps $A \to B$.

(b) Do the same for $B \to A$.

Q3 (4 marks): Let Y be the space formed from a solid triangle $X = \Delta^2$ by identifying the three vertices to a single point. Using only what we have covered in class, compute the reduced singular homology groups $\tilde{H}_n(Y)$, for all n.

[Hint: You can use Theorem 2.13, even though we haven't proved it yet.]