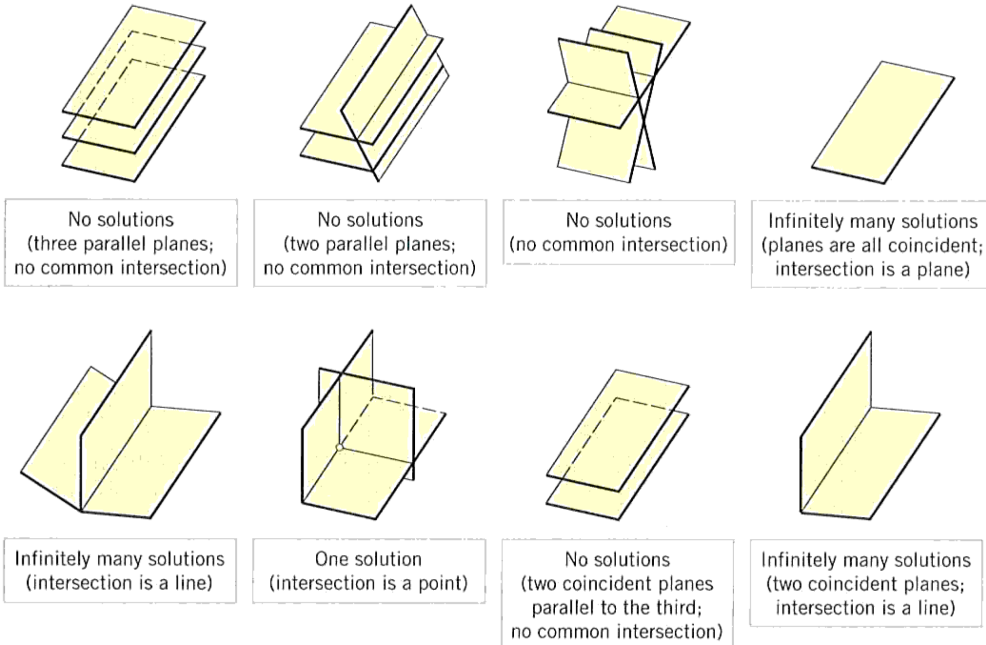
Section 2.1: Introduction to systems of linear equations



Elementary row operations.

1. Multiply a row through by a nonzero constant.
2. Interchange two rows.
3. Add a multiple of one row to another.

Section 2.2: Solving linear systems by row reduction

Row echelon form and reduced row echelon form.

1. If a row does not consist entirely of zeroes, then the first non-zero number in the row is a 1. We call this a leading 1.
2. If there are any rows that consist entirely of zeroes, then they are grouped together at the bottom of the matrix.
3. In any two successive rows that do not consist entirely of zeroes, the leading 1 in the lower row occurs further to the right than the leading 1 in the higher row.
4. (Reduced REF only) Each column that contains a leading 1 has zeroes everywhere else.

Gaussian and Gauss–Jordan eliminiation.

Step 1 Locate the leftmost column that does not consist entirely of zeroes.

Step 2 Interchange the top row with another row, if necessary, to bring a non-zero entry to the top of the column found in Step 1.

Step 3 Let a be the entry that is now at the top of the column found in Step 1. Multiply the first row by $1/a$ in order to introduce a leading 1.

Step 4 Add suitable multiples of the top row to the rows below so that all entries below the leading 1 become zeroes.

Step 5 Now cover the top row in the matrix and begin with Step 1 applied to the submatrix that remains. Continue in that way until the entire matrix is in row echelon form.

Step 6 (Gauss–Jordan only) Beginning with the last non-zero row and working upward, add suitable multiples of each row to the rows above to introduce zeroes above the leading 1's.

Section 3.2: Inverses; algebraic properties of matrices

Theorem 3.2.1. *For a, $b \in \mathbb{R}$ and matrices A, B, C we have the following identities whenever the expressions are defined:*

(*a*) $A + B = B + A$
(*b*) $A + (B + C) = (A + B) + C$
(*c*) $(ab)A = a(bA)$
(*d*)+(*e*) $(a \pm b)A = aA \pm bA$
(*f*)+(*g*) $a(A \pm B) = aA \pm aB$

Theorem 3.2.2. *For $a \in \mathbb{R}$ and matrices A, B, C we have the following identities whenever the expressions are defined:*

(*a*) $A(BC) = (AB)C$
(*b*)+(*d*) $A(B \pm C) = AB \pm AC$
(*c*)+(*e*) $(B \pm C)A = BA \pm CA$
(*f*) $a(BC) = (aB)C = B(aC)$

Theorem 3.2.3. *For $c \in \mathbb{R}$, a matrix A and a zero matrix 0 of appropriate size, we have:*

(*a*)+(*b*) $A \pm 0 = 0 + A = A$
(*c*) $A - A = A + (-A) = 0$
(*d*) $0\,A = 0$
(*e*) *If $cA = 0$, then $c = 0$ or $A = 0$.*

Theorem 3.2.10. *For matrices A and B of the appropriate sizes:*

(*a*) $(A^T)^T = A$
(*b*)+(*c*) $(A \pm B)^T = A^T \pm B^T$
(*d*) $(kA)^T = kA^T$
(*e*) $(AB)^T = B^T A^T$

Theorem 3.2.12. *If A and B are square matrices of the same size, then:*

(*a*) $\mathrm{tr}(A^T) = \mathrm{tr}(A)$
(*b*) $\mathrm{tr}(cA) = c\,\mathrm{tr}(A)$
(*c*)+(*d*) $\mathrm{tr}(A \pm B) = \mathrm{tr}(A) \pm \mathrm{tr}(B)$
(*e*) $\mathrm{tr}(AB) = \mathrm{tr}(BA)$

Section 3.3: Elementary matrices; a method for finding A^{-1}

Theorem 3.3.9 (Unifying Theorem). *If A is an $n \times n$ matrix, then the following statements are equivalent.*

(*a*) *The reduced row echelon form of A is I_n.*
(*b*) *A is expressible as a product of elementary matrices.*
(*c*) *A is invertible.*
(*d*) *$A\mathbf{x} = \mathbf{0}$ has only the trivial solution.*
(*e*) *$A\mathbf{x} = \mathbf{b}$ is consistent for every vector $\mathbf{b}$ in $\mathbb{R}^n$.*
(*f*) *$A\mathbf{x} = \mathbf{b}$ has exactly one solution for every vector $\mathbf{b}$ in $\mathbb{R}^n$.*