8 marks

1. Find the value of t so that

$$\mathbf{v} = \begin{bmatrix} 3 \\ 2 \\ -1 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} 1 \\ -3/2 \\ t \\ 1 \end{bmatrix}$$

are orthogonal. Remember to justify your answer. Answers without justification will not recieve credit.

8 marks 2. Solve the following linear system by row reduction.

$$\begin{array}{rcrcrcr} 3x & - & y & + & 2z & = & 12 \\ x & + & 2y & + & 3z & = & 11 \\ 2x & - & 2y & + & z & = & 2 \end{array}$$

Remember to justify your answer. Answers without justification will not recieve credit.

7 marks

3. Let $A = \begin{bmatrix} 4 & 1 & 2 & 1 \\ 3 & 0 & 1 & 6 \\ 5 & 7 & 9 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 & 2 & 1 \\ 3 & 0 & 1 & 6 \\ 8 & 7 & 10 & 14 \end{bmatrix}$. Find an elementary matrix E so that $EA = B$. Remember to justify your answer. Answers without justification will not recieve credit.

7 marks

4. In both these questions, remember to justify your answer. Answers without justification will not receive credit.

   (a) Show that the collection V of vectors in R^2 of the form $\begin{bmatrix} t \\ 2t \end{bmatrix}$, where t is an arbitrary real number, is a subspace.

   (b) Show that the collection W of vectors of the form $(a+1, b, a)$ in R^3 with with a and b arbitrary real numbers is not a subspace.

7 marks 5. Find the determinant of the matrix

$$\begin{bmatrix} 6 & -2 & -4 & 4 \\ 3 & -3 & -6 & 1 \\ -12 & 8 & 21 & -8 \\ -6 & 0 & -10 & 7 \end{bmatrix}.$$

Remember to show your work.

7 marks

6. In both the following questions,remember to show all work.

(a) Find the characteristic polynomial of $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$.

(b) Find the eigenvalues of A.

4 marks 7. Remember to show all work. Let $T : R^3 \rightarrow R^3$ be the linear transformation $T(x, y, z) = (x - y, z, y - x)$.

(a) Find a set of vectors that spans the kernel of T.

(b) Find a set of vectors that spans the range of T.

4 marks 8. Let $T : R^2 \to R^2$ be the linear transoformation

$$T(x, y) = (x - y, 2y).$$

Let $U : R^2 \to R^2$ be rotation anti-clockwise by $45^\circ = \frac{\pi}{4}$ rad. Find the standard matrix of $U \circ T$. Remember to justify your answer. Answers without justification will not recieve credit.

4 marks

9. It is known that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a skew-symmetric matrix with $\det A = -4$. Further $b > 0$. Find a, b, c, d. Remember to justify your answer. Answers without justification will not recieve credit.

7 marks 10.

Find the general equation of the plane in R^3 passing through the point $(1, 1, 1)$ and normal to the vector $(1, 2 - 2)$. Remember to justify your answer. Answers without justification will not recieve credit.

7 marks

11. Which of the following statements is true or false? There is no need to justify your answer.

    Circle T for true or F for false. If you do not know the answer do not guess; leave the question unanswered.

    You will lose 1.5 marks for each incorrectly answered question.

    (a) Every spanning set of R^3 has at least 3 vectors. **T** **F**

    (b) Suppose S is a collection of vectors in a subspace V. If span $S = V$ then S contains a basis for V. **T** **F**

    (c) Suppose T is a linear transformation. If $\mathbf{v}_1$ and $\mathbf{v}_2$ are in $\ker(T)$ then so is $\mathbf{v}_1 - \mathbf{v}_2$. **T** **F**

    (d) If A is a square matrix then $\det A^3 = 3 \det A$ **T** **F**

    (e) If $\mathbf{v}_1$ and $\mathbf{v}_2$ are 2 vectors in R^3 one can always find a vector $\mathbf{v}_3$ so that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of R^3. **T** **F**

    (f) The only eigenvalue of an invertible matrix is 1. **T** **F**

    (g) The columns of an invertible $n \times n$ matrix span R^n. **T** **F**

7 marks

12. In both these questions, remember to justify your answer. Answers without justification will not receive credit.

 (a) Show that the transformation $T : R^2 \to R^2$ given by $T(x, y) = (x + y, 2y)$ is linear.

 (b) Show that the transformation $T : R^2 \to R^2$ given by $T(x, y) = (x + 1, 2y)$ is not linear.

7 marks 13. Is the linear transformation $T : R^2 \to R^2$ with standard matrix

$$M = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \end{bmatrix}$$

an orthogonal operator? You must justify your answer, answers without justifiction will not receive marks.

7 marks

14. Show that the collection of vectors

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

form a basis of R^3. You must justify your answer, answers without justifiction will not receive marks.

3 marks 15. Set

$$
\begin{aligned}
f_1 &= e_1 \\
f_2 &= e_1 + 2e_2 \\
f_3 &= e_1 + 2e_2 + 3e_3 \\
&\vdots \\
f_i &= e_1 + 2e_2 + \cdots + ie_i
\end{aligned}
$$

It is known that A is a 7×7 matrix that has f_i as an eigenvector with eigenvalue 2^i for $1 \leq i \leq 7$. Find A^{-1}. You must justify your answer, answers without justifiction will not receive marks.

3 marks 16. It is known that A is a 4×4 matrix. Some entries of A and the adjoint of A are known and are displayed below.

$$A = \begin{bmatrix} a & 10 & 4 & 3 \\ -1 & -1 & -1 & 0 \\ -1 & -2 & 2 & -2 \\ c & 2 & 1 & d \end{bmatrix} \qquad \mathrm{Adj}(A) = \begin{bmatrix} -2 & -14 & * & * \\ 3 & 7 & * & * \\ -1 & 0 & * & * \\ b & 0 & * & * \end{bmatrix}.$$

Find the values of a, b, c, d and the determinant of A. Here the *'s have some fixed value that is unkown to you. You must justify your answer. Answers without justifiction will not receive marks.

3 marks 17. A is a matrix with $\text{rref}(A) = \begin{bmatrix} 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Further $A \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \mathbf{b}$. Is $\begin{bmatrix} 14 \\ 8 \\ 13 \\ -1 \end{bmatrix}$ a solution of the linear system $A\mathbf{x} = \mathbf{b}$? You must justify your answer. Answers without justifiction will not receive marks.

Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENTS OF APPLIED MATHEMATICS AND MATHEMATICS

Linear Algebra 040b Midterm Examination

Saturday, March 8, 2008 7:00 p.m. - 10:00 p.m.

INSTRUCTIONS

1. DO NOT UNSTAPLE THE BOOKLET.
2. CALCULATORS AND NOTES ARE NOT PERMITTED.
3. SHOW ALL YOUR WORK.
4. Questions start on Page 1 and continue to Page 17. Be sure that your booklet is complete. Questions are printed on both sides of the paper.
5. TOTAL MARKS = 100.

Student Number (**Print**)

Student's Name (**Print**)

FOR GRADING ONLY

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