Math 1600A Lecture 5, Section 002

Announcements:

More texts, solutions manuals and packages coming soon, possibly today.

Read Section 1.3, the Exploration on cross products and Section 1.4 (just the part on code vectors) for next class. Work through recommended homework questions. Scans of **all** of sections 1.1, 1.2 and 1.3 are available from the course home page.

Tutorials start **this week**, and include a **quiz** covering until the end of Section 1.2. It does not cover Section 1.3 or the Exploration after Section 1.2.
The quizzes last 20 minutes, and are at the end of the tutorial, so you have time for questions at the beginning.
Questions *similar* to homework questions, but may be slightly different. There are some true/false questions.
You must write in the tutorial you are registered in.
Different sections have different quizzes, but it is still considered an academic offense to share information about quizzes.

Office hour: today, 12:30-1:30, MC103B.
Also, if you can't make it to my office hours, feel free to attend Hugo Bacard's office hours.

Help Centers Monday-Friday 2:30-6:30 in MC 106 starting Thursday.

Lecture notes (this page) available from course web page by clicking on our class times.

New material

[These notes are a summary of the material, which will be supplemented by lots of diagrams on the whiteboard.]

Section 1.3: Lines and planes in $\mathbb{R}^2$ and $\mathbb{R}^3$

We study lines and planes because they come up directly in applications, but also because the solutions to many other types of problems can be expressed using the language of lines and planes.

Lines in $\mathbb{R}^2$ and $\mathbb{R}^3$

Given a line ℓ, we want to find equations that tell us whether a point (x, y) or (x, y, z) is on the line. We'll write $\vec{x} = [x, y]$ or $\vec{x} = [x, y, z]$ for the position vector of the point, so we can use vector notation.

The **vector form** of the equation for ℓ is:

$$\vec{x} = \vec{p} + t\vec{d}$$

where $\vec{p}$ is the position vector of a point on the line, $\vec{d}$ is a vector parallel to the line, and $t \in \mathbb{R}$.
This is concise and works in $\mathbb{R}^2$ and $\mathbb{R}^3$.

If we expand the vector form into components, we get the **parametric form** of the equations for ℓ:

$$\begin{aligned} x &= p_1 + td_1 \\ y &= p_2 + td_2 \\ (z &= p_3 + td_3 \quad \text{if we are in } \mathbb{R}^3) \end{aligned}$$

Lines in $\mathbb{R}^2$

For a line in $\mathbb{R}^2$, there are additional ways to describe a line.

The **normal form** of the equation for ℓ is:

$$\vec{n} \cdot (\vec{x} - \vec{p}) = 0 \quad \text{or} \quad \vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p},$$

where $\vec{n}$ is a vector that is *normal* = *perpendicular* to ℓ.

If we write this out in components, with $\vec{n} = [a, b]$, we get the **general form** of the equation for ℓ:

$$ax + by = c,$$

where $c = \vec{n} \cdot \vec{p}$. When $b \neq 0$, this can be rewritten as $y = mx + k$, where

$m = -a/b$ and $k = c/b$.

Note: All of these simplify when the line goes through the origin, as then you can take $\vec{p} = \vec{0}$.

Example: Find all four forms of the equations for the line in $\mathbb{R}^2$ going through $A = [1,1]$ and $B = [3,2]$.

Note: None of these equations is *unique*, as $\vec{p}$, $\vec{d}$ and $\vec{n}$ can all change. The general form is closest to being unique: it is unique up to an overall scale factor.

Lines in $\mathbb{R}^3$

Most of the time, one uses the vector and parametric forms above. But there is also a version of the normal and general forms. To specify the direction of a line in $\mathbb{R}^3$, it is necessary to specify **two** non-parallel normal vectors $\vec{n}_1$ and $\vec{n}_2$. Then the **normal form** is

$$\begin{aligned} \vec{n}_1 \cdot \vec{x} &= \vec{n}_1 \cdot \vec{p} \qquad \text{typo in book in Table 1.3:} \\ \vec{n}_2 \cdot \vec{x} &= \vec{n}_2 \cdot \vec{p} \qquad \text{there should be no subscripts on } \vec{p} \end{aligned}$$

When expanded into components, this gives the **general form**:

$$\begin{aligned} a_1 x + b_1 y + c_1 z &= d_1, \\ a_2 x + b_2 y + c_2 z &= d_2. \end{aligned}$$

Since *both* equations must be satisfied, we'll in just a second that this can be interpreted as the intersection of two planes.

Question: What are the pros and cons of the different ways of describing a line?

Planes in $\mathbb{R}^3$

Normal form:

$$\vec{n} \cdot (\vec{x} - \vec{p}) = 0 \quad \text{or} \quad \vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}.$$

This is *exactly* like the normal form for the equation for a line in $\mathbb{R}^2$.
When expanded into components, it gives the **general form**:

$$ax + by + cz = d,$$

where $\vec{n} = [a, b, c]$ and $d = \vec{n} \cdot \vec{p}$.

Note: You can read off $\vec{n}$ from the general form. Two planes are parallel if and only if their normal vectors are parallel.

A plane can also be described in **vector form**. You need to specify a point $\vec{p}$ in the plane as well as two vectors $\vec{u}$ and $\vec{v}$ which are parallel to the plane but not parallel to each other.

$$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$$

When expanded into components, this gives the **parametric equations** for a plane:

$$\begin{aligned} x &= p_1 + su_1 + tv_1 \\ y &= p_2 + su_2 + tv_2 \\ z &= p_3 + su_3 + tv_3. \end{aligned}$$

Table 1.3 in the text summarizes this nicely (except for the one typo mentioned above).

It may seem like there are lots of different forms, but really there are two: vector and normal, and these can be expanded into components to give the parametric and general forms.

Example: Find all four forms of the equations for the plane in $\mathbb{R}^3$ which goes through the point $P = (1, 2, 0)$ and has normal vector $\vec{n} = [2, 1, -1]$.

If you get parallel vectors $\vec{u}$ and $\vec{v}$, you need to try again.