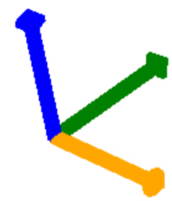
Math 1600A Lecture 1, Section 2, 5 Sep 2014

Announcements:

Torus knot with tangent vector (brown), normal vector (green) and binormal vector (blue)



Discuss syllabus. Some key points:

- This course is cumulative and gets tough. Keep up!
- Before next class, **read "To the student", Section 1.0 and Section 1.1 in the text**. In general, **read the text**.
- Do **exercises** as we cover the material, and again before quizzes and exams.
- Answers to odd exercises are at the end of the text; solutions are in the study guide, but aren't always clear.
- Print a copy of the syllabus for reference, and don't e-mail me questions that are answered on it.
- Choice of 1229 vs 1600: 1229 covers less material, is aimed at social science students, and has fewer prerequisites. But 1600 is required for many programs. 1229 can be taken before 1600, but they can't be taken at the same time. See a counsellor if needed.
- **Questions welcome** at any time! Are there any now?

O-week event: "*What is Mathematics?*" Rescheduled to Friday (today), 3pm-4pm, Middlesex College 107, followed by free pizza and pop!

New material

Section 1.1: The Geometry and Algebra of Vectors

scalar	**vector**
real valued quantity	magnitude and direction

speed: 10 m/s	velocity: 10 m/s north	$\uparrow \mathrm{10\ m/s}$
temperature: 10 C	force: 10 Newtons up	$\uparrow \mathrm{10\ N}$
distance: 10 m	displacement: 10 m east	$\xrightarrow{\mathrm{10\ m}}$

Vectors in the plane

If A and B are points in the plane, then $\overrightarrow{AB}$ denotes the vector from A to B. The point A is called the **initial point** and B is called the **terminal point**. (Sketch on board.)

The components of a vector are its horizontal and vertical displacements.

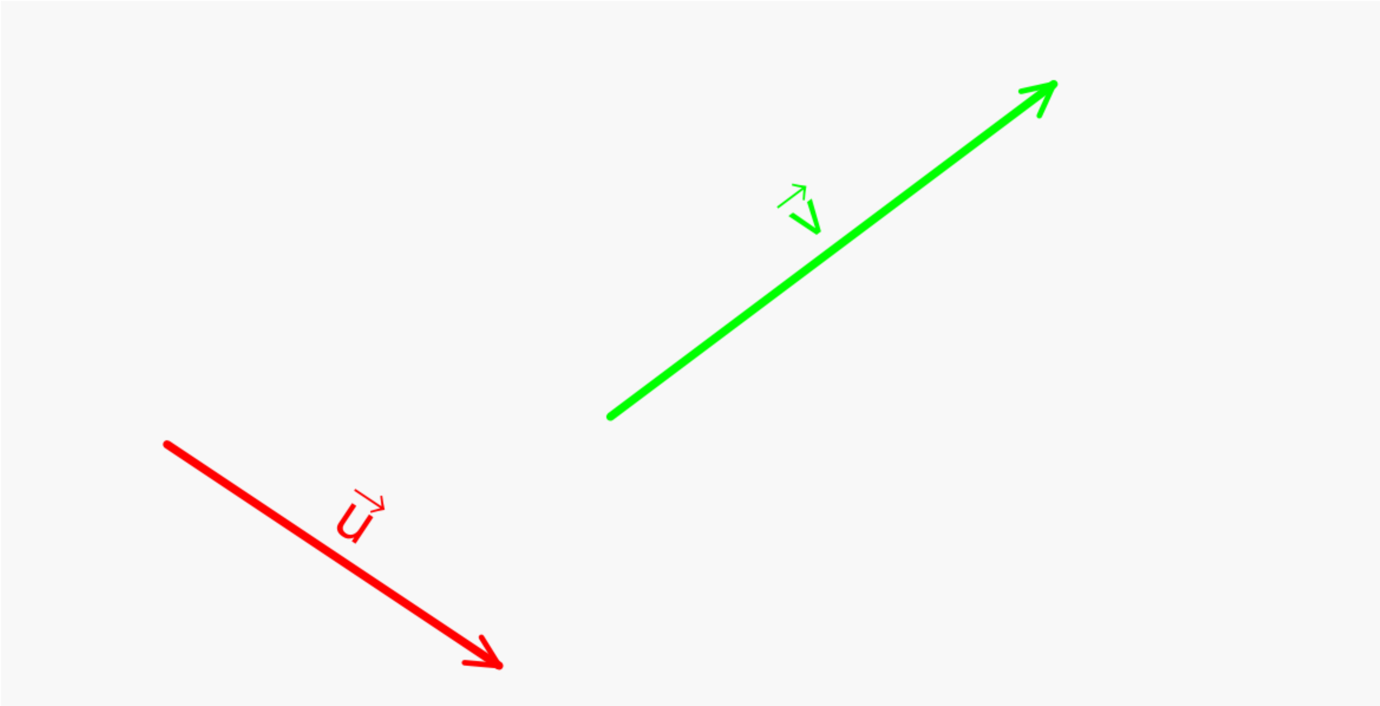
For example, if $A = (2, 4)$ and $B = (5, 6)$, then the components of $\overrightarrow{AB}$ are $5 - 2 = 3$ and $6 - 4 = 2$. We write $\overrightarrow{AB} = [3, 2] = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (order matters).

The overall position of a vector does not matter. Two vectors are considered **equal** if they have the same length and direction, or equivalently if their components are equal. For example, if $C = (3, 2)$ and $O = (0, 0)$ is the origin, then $\overrightarrow{AB} = \overrightarrow{OC}$.

We write $\mathbb{R}^2$ for the set of all vectors with two real numbers as components. So $[3, 2]$, $[-\pi, 7/2]$ and $\vec{0} = [0, 0]$ are all vectors in $\mathbb{R}^2$.

New vectors from old

Vector addition: triangle rule: To add $\vec{u}$ and $\vec{v}$, translate them so the initial point of $\vec{v}$ equals the terminal point of $\vec{u}$, and draw an arrow from the initial point of $\vec{u}$ to the terminal point of $\vec{v}$:



[Drag midpoint to translate vectors, or endpoints to change vectors. Press "p" to toggle parallelogram rule and "r" to resize canvas.]

Paralleogram rule: Explain with the applet.

Algebraically, to add vectors, you add the corresponding components, so for $\vec{u} = [u_1, u_2]$ and $\vec{v} = [v_1, v_2]$ we have

$$\vec{u} + \vec{v} := [u_1 + v_1, u_2 + v_2]$$

Scalar multiplication: for $c \in \mathbb{R}$ and $\vec{v} = [v_1, v_2]$, we define

$$c\vec{v} = c[v_1, v_2] := [cv_1, cv_2].$$

Geometrically, this scales the length by the absolute value $|c|$ of c, and reverses the direction if $c < 0$. (Sketch on board.) We say that $\vec{v}$ and $\vec{w}$ are **parallel** if $\vec{v} = c\vec{w}$ or $\vec{w} = c\vec{v}$ for some $c \in \mathbb{R}$. (Note that $c = 0$ and $c < 0$ are permitted.)

We refer to real numbers as **scalars**.

Negative: We define $-\vec{v} := (-1)\vec{v} = [-v_1, -v_2]$.

Subtraction: We define $\vec{u} - \vec{v} := \vec{u} + (-\vec{v}) = [u_1 - v_1, u_2 - v_2]$.

Zero vector: We define $\vec{0} = [0, 0]$.

Vectors in $\mathbb{R}^3$

In 3-space, a vector has three components, giving its displacements parallel to the x, y and z axes: $\vec{v} = [v_1, v_2, v_3]$. The collection of such vectors is denoted $\mathbb{R}^3$. All of the operations we have discussed extend to $\mathbb{R}^3$. The text gives some geometrical illustrations.

Vectors in $\mathbb{R}^n$

It is important for applications to be able to deal with vectors with more than three components. We write $\mathbb{R}^n$ for the set of ordered n-tuples of real numbers. For example, $[1, 0, 4, 3, 2]$ is a vector in $\mathbb{R}^5$.

While we can't visualize such vectors geometrically, the algebraic definitions extend immediately to this case:

If $\vec{u} = [u_1, u_2, \ldots, u_n]$ and $\vec{v} = [v_1, v_2, \ldots, v_n]$ and $c \in \mathbb{R}$, then

$$\vec{u} + \vec{v} := [u_1 + v_1,\ u_2 + v_2,\ \ldots,\ u_n + v_n]$$

E.g. $[3, 2, 1, 0] + [1, 0, -1, 4] = [4, 2, 0, 4]$.

$$c\vec{u} = c[u_1, \ldots, u_n] := [cu_1,\ \ldots,\ cu_n]$$

E.g. $2[1, 2, 3, 4, 5] = [2, 4, 6, 8, 10]$.

$$-\vec{u} := (-1)\vec{u} = [-u_1, -u_2, \ldots, -u_n]$$

E.g. $-[1, 2, 3, 4, 5] = [-1, -2, -3, -4, -5]$.

$$\vec{u} - \vec{v} := \vec{u} + (-\vec{v}) = [u_1 - v_1,\ \ldots,\ u_n - v_n]$$

E.g. $[1, 2, 3, 4, 5] - [1, 0, 2, 1, 1] = [0, 2, 1, 3, 4]$.

$$\vec{0} := [0, 0, \ldots, 0]$$