Page:	2	3	4	5	6	7	8	9	10	11	Total
Marks:	8	8	6	10	9	5	4	7	5	7	$69 + 1$
Score:											

Name (print): ______________________________

Signature: ______________________________

UWO ID number: ______________________________

CIRCLE THE NUMBERS OF YOUR **LECTURE** AND **LAB** SECTIONS:

001	MWF 12:30	Stu Rankin
002	MWF 10:30	Dan Christensen

003	Thu 1:30	Allen O'Hara	006	Thu 10:30	Javad Rastegari Koopaei
004	Thu 12:30	Jessica Yau	007	Wed 10:30	Javad Rastegari Koopaei
005	Thu 2:30	Jessica Yau	008	Wed 9:30	Chandra Rajamani

THE UNIVERSITY OF WESTERN ONTARIO
DEPARTMENT OF MATHEMATICS

MATHEMATICS 1600B MIDTERM EXAMINATION
1 March 2014 6:30-9:30 PM

INSTRUCTIONS:

1. This exam is 12 pages long. There are 12 questions. **Check that your exam is complete.**
2. All questions must be answered in the space provided. Indicate your answer clearly. Should you need extra space, a blank page is provided at the end of the booklet. Coloured paper is for rough work only.
3. Show all your of your work and explain your answers fully. Unjustified, irrelevant or illegible answers will receive little or no credit.
4. Do not unstaple the exam booklet.
5. **No aids are permitted. In particular, calculators, cell phones, ipods, etc. are not allowed and may be confiscated.**
6. If not stated otherwise, all vectors and equations involve real numbers.
7. Many answers can be easily checked. **You get 1 mark if we see that you checked at least one answer!**

1. For each of the following statements, circle **T** if the statement is always true and **F** if it can be false. **Give a brief justification for your answer.**

[2] (a) Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be nonzero vectors in $\mathbb{R}^3$. If $\vec{u}$ and $\vec{v}$ are each orthogonal to $\vec{w}$, then $2\vec{u} - 3\vec{v}$ is orthogonal to $\vec{w}$. **T** **F**

[2] (b) The planes $3x - 2y + z = 2$ and $-9x + 6x - 3z = 4$ are parallel. **T** **F**

[2] (c) The matrix $\begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$ has rank 2. **T** **F**

[2] (d) Let A denote the coefficient matrix of a system of 4 linear equations in 3 unknowns. Then this system has a unique solution. **T** **F**

[2] (e) Let A be a 2×3 matrix. Then the column vectors of A are linearly dependent.

T **F**

[2] (f) Let A and B be 2×2 matrices. Then $AB = BA$. **T** **F**

[2] (g) The matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is invertible. **T** **F**

[2] (h) The set $S = \{[x, y] \in \mathbb{R}^2 \mid xy = 0\}$ is a subspace of $\mathbb{R}^2$. **T** **F**

[2] 2. Suppose that $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^4$ are such that $\vec{u} \cdot \vec{v} = -2$, $\vec{u} \cdot \vec{w} = 2$, $\|\vec{v}\| = 3$, and $\vec{v} \cdot \vec{w} = 1$. Compute $(2\vec{u} - \vec{v}) \cdot (3\vec{v} + \vec{w})$.

[2] 3. (a) Find the unit vector in the same direction as $\vec{x} = [2, 1, 2]$.

[2] (b) Find the cosine of the angle θ between $\vec{x} = [2, 1, 2]$ and $\vec{y} = [3, -1, 2]$.

[4] 4. Let $Q = (2,3)$, $P = (-1,7)$, and $\vec{d} = [1,2]$. Find the distance from the point Q to the line through P in the direction of $\vec{d}$.

[3] 5. (a) Find a normal vector for the plane in $\mathbb{R}^3$ that passes through $P = (1,2,1)$, $Q = (-1,1,0)$, and $R = (2,1,3)$.

[3] (b) Find a general equation for the plane in (a).

[3] 6. Find a vector equation for the line of intersection of the planes $2x-2y+2z=4$ and $2x-y+3z=1$.

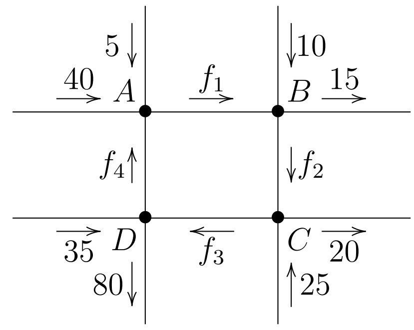
7. Recall that the Universal Product Code (UPC) uses code words in $\mathbb{Z}_{10}^{12}$ and has check vector $c=[3,1,3,1,3,1,3,1,3,1,3,1]$.

[3] (a) Is the vector $[0,1,3,2,4,3,5,4,6,5,7,6]$ a valid UPC? Explain.

[3] (b) Find the missing digit y in the UPC $[0,1,2,1,3,9,y,5,0,7,3,4]$.

[5] 8. Determine whether the vector $\vec{v} = \begin{bmatrix} 12 \\ 3 \\ 4 \end{bmatrix}$ is in the span of the vectors $\vec{u}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\vec{u}_3 = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix}$. If it is, find a way to express $\vec{v}$ as a linear combination of $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_3$.

9. Consider the pictured network of water pipes, where the flows are measured in litres/minute. In this question, flows can be positive or negative.



[2] (a) Set up a system of linear equations for the possible flows f_1, f_2, f_3 and f_4.

[2] (b) Solve the system of equations.

10. Let $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & 0 \\ -1 & -2 & 3 \end{bmatrix}$.

[4] (a) Determine whether A is invertible, and if so, find A^{-1}, showing your work.

[3] (b) Solve the system $A\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ for $\vec{x}$.

11. Let
$$A = \begin{bmatrix} 2 & 4 & 2 & 0 & 2 \\ 3 & 5 & 3 & -2 & 7 \\ 4 & 7 & 4 & -2 & 8 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 1 & 0 & 1 & -4 & 9 \\ 0 & 1 & 0 & 2 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Given that R is the reduced row-echelon form of A, compute each of the following. Explain briefly.

[1] (a) rank(A).

[2] (b) A basis for row(A).

[2] (c) A basis for col(A).

Continued on next page...

Continued from previous page...

[3] (d) A basis for null(A).

[4] 12. Let A, B and X be invertible $n \times n$ matrices. Solve the following matrix equation for X in terms of A and B. Show your steps, and simplify your answer.

$$(BX + A)^T = A + A^T + B^T$$

Use this page if you need extra space for your work.

Did you write your name and student ID on the first page?

Did you give full explanations and show all of your work?

Did you check your answers?