Page:	2	3	4	5	6	7	8	Total
Marks:	6	7	3	6	6	7	5	40
Score:								

Name (print): ______________________________

Signature: ______________________________

UWO ID number: ______________________________

CIRCLE THE NUMBERS OF YOUR **LECTURE** AND **LAB** SECTIONS:

001	MWF 8:30	Hugo Bacard
002	MWF 10:30	Dan Christensen

003	Wed 9:30	Youlong Yan	006	Wed 3:30	Javad Rastegari Koopaei
004	Thu 2:30	Allen O'Hara	007	Thu 12:30	Gaohong Wang
005	Thu 11:30	Jason Haradyn	008	Wed 11:30	Jason Haradyn

THE UNIVERSITY OF WESTERN ONTARIO
DEPARTMENT OF MATHEMATICS

MATHEMATICS 1600A MIDTERM EXAMINATION 2
7 November 2013 7:00–8:30 PM

INSTRUCTIONS:

1. This exam is 9 pages long. There are 7 questions. **Check that your exam is complete.**
2. All questions must be answered in the space provided. Indicate your answer clearly. Should you need extra space, a blank page is provided at the end of the booklet.
3. Show all your of your work and explain your answers fully. Unjustified, irrelevant or illegible answers will receive little or no credit.
4. Do not unstaple the exam booklet.
5. **No aids are permitted. In particular, calculators, cell phones, ipods, etc. are not allowed and may be confiscated.**
6. If not stated otherwise, all vectors and equations involve real numbers.

1. For each of the following statements, circle **T** if the statement is always true and **F** if it can be false. Give a one-sentence justification for your answer.

[2] (a) The set $S = \{[x, y] \in \mathbb{R}^2 \mid x + y \geqslant -1\}$ is a subspace of $\mathbb{R}^2$. **T** **F**

[2] (b) Let A be a square matrix. If the column vectors of A are linearly independent, then the row vectors of A are linearly independent too. **T** **F**

[2] (c) Let M be a 3×2 matrix. Then the column vectors of M are linearly dependent. **T** **F**

[2] (d) If A and B are both symmetric $n \times n$ matrices and c and d are any scalars, then $cA + dB$ is also symmetric.

T **F**

[2] (e) The dimension of span$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ is 3.

T **F**

[3] 2. Let A, B and X be invertible $n \times n$ matrices. Solve the following matrix equation for X in terms of A and B. Show your steps, and simplify your answer.

$$A(X^T + B) = (B + A)B$$

[3] 3. Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

4. Let

$$A = \begin{bmatrix} 0 & 1 & -4 & -1 & 1 \\ 0 & 3 & -12 & 0 & 9 \\ 0 & 2 & -8 & 2 & 10 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 0 & 1 & -4 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Given that R is the reduced row-echelon form of A, compute each of the following. Explain briefly.

[1] (a) rank(A).

[1] (b) nullity(A).

[2] (c) A basis for row(A).

[2] (d) A basis for col(A).

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[3] (e) A basis for null(A).

[3] 5. Let A be a 2×4 matrix. What are the possible values of the rank and nullity of A? Explain.

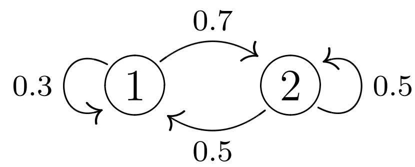
6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that

$$T(\vec{e}_1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad T(\vec{e}_2) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

[2] (a) Find the standard matrix $[T]$ of T.

[3] (b) Determine whether T is invertible. If so, find the standard matrix of T^{-1}.

[2] (c) Find the standard matrix of $T^2 = T \circ T$.

7. Consider the following Markov process: 

[2] (a) Write down the transition matrix P for this Markov process.

[3] (b) Find a steady state probability vector for P.

Use this page if you need extra space for your work.

Did you write your name and student ID on the first page?
Did you give full explanations and show all of your work?