Question:	1	2	3	4	5	6	7	8	9	Total
Marks:	4	2	2	2	4	4	4	5	5	32
Score:										

Name (print): ______________________________

Signature: ______________________________

UWO ID number: ______________________________

Circle the numbers of your section and lab section in the tables below:

001	MWF 12:30	Matthias Franz
002	MWF 11:30	Derek Krepski

003	Thu 1:30	A. Ghorbanpour	007	Wed 10:30	J. Rastegari Koopaei
004	Thu 12:30	A. Ghorbanpour	008	Wed 9:30	Y. Yan
005	Thu 2:30	J. Haradyn	009	Wed 10:30	G. Wang
006	Thu 10:30	J. Rastegari Koopaei			

THE UNIVERSITY OF WESTERN ONTARIO
DEPARTMENT OF MATHEMATICS

MATHEMATICS 1600B MIDTERM EXAMINATION
07 March 2013 7:00–8:30 PM

INSTRUCTIONS:

1. This exam is 7 pages long. It is printed double-sided. There are 9 questions.
2. All questions must be answered in the space provided. Indicate your answer clearly. Should you need extra space, a blank page is provided at the end of the booklet.
3. Show all of your work and explain your answers fully. Unjustified, irrelevant or illegible answers will receive little or no credit.
4. Do not unstaple the exam booklet.
5. **No aids are permitted. In particular, calculators, cell phones, ipods etc. are not allowed and may be confiscated.**
6. If not stated otherwise, all vectors and equations involve real numbers.
7. Unless instructed otherwise, you must give all complex numbers in your final answer in Cartesian (real + imaginary) or polar form. Likewise, in your final answers you must give all numbers in $\mathbb{Z}_n$ as a number between 0 and $n-1$.

1. For each of the following statements, circle **T** if the statement is always true and **F** if it can be false. Give a one-sentence justification for your answer.

   (a) (2 marks) If $z^2 + w^2 = 0$, where z and w are complex numbers, then it must be that both $z = 0$ and $w = 0$.

   T **F**

   (b) (2 marks) If the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are linearly dependent, then $\mathbf{u}$ is a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

   T **F**

2. (a) (1 mark) Give an example of a matrix A such that the columns of A are linearly independent, while the rows of A are linearly dependent.

   (b) (1 mark) Give an example of two square matrices A and B such that $(A+B)^2 \neq A^2 + 2AB + B^2$.

3. (2 marks) Find the complex number z satisfying $(1+i)z + (1-i)^2 = 6 - 2i$. (Express your answer in the form $z = a + bi$.)

4. (2 marks) Assume that X, A and B are square matrices of the same size, and that A and B are invertible. Solve the following matrix equation for X.

$$B^{-1}A^{-1}(X + A) = BA^{-1} + B^{-1}$$

5. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ be a matrix with coefficients in $\mathbb{Z}_3$.

   (a) (2 marks) Compute $A^2 + A$.

   (b) (2 marks) Find all 2×2 matrices X (with coefficients in $\mathbb{Z}_3$) such that $AX = 0$.

6. Let $A = \begin{bmatrix} -2 & 1 & 2 \\ -1 & 2 & 2 \\ -2 & 4 & 3 \end{bmatrix}$.

(a) (3 marks) Calculate A^{-1}.

(b) (1 mark) Find z in the system of linear equations below. (You may wish to use your answer from the previous part.)

$$\begin{array}{rrrcl} -2x & +y & +2z & = & 5 \\ -x & +2y & +2z & = & -3 \\ -2x & +4y & +3z & = & 1 \end{array}$$

7. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

(a) (3 marks) Determine whether $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a spanning set for $\mathbb{R}^3$.

(b) (1 mark) Is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ linearly independent? Explain.

8. Let

$$A = \begin{bmatrix} 1 & -2 & 1 & 4 & -1 & 9 \\ 1 & -2 & 2 & 7 & -1 & 11 \\ -1 & 2 & -1 & -4 & 2 & -12 \\ 1 & -2 & -1 & -2 & -2 & 8 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 1 & -2 & 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Given that R is the reduced row-echelon form of A, find the following, explaining your work.

(a) (1 mark) A basis for row(A)

(b) (2 marks) A basis for col(A)

(c) (2 marks) A basis for null(A)

9. For a complex number $z = a + bi$, consider the real 2×2 matrix $A_z = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. For example, in the case $z = i$ one has $A_i = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

(a) (1 mark) If $z = e^{(3\pi/4)i}$, find A_z.

(b) (2 marks) Compute $A_i^{100} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^{100}$.

(c) (2 marks) Compute $(A_z)^{-1}$ for $z \neq 0$.

Use this page if you need extra space for your work.

Did you write your name and student ID on the first page?
Did you give full explanations and show all of your work?